Advantages of a Non-Force Free Approach to Modeling Magnetic Clouds

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- The Physics of Force-Free Flux Ropes
- The Bessel Function Solution
- The Non Force Free Model of Mulligan et al.
- Analyses of Pioneer Venus data
- Summary and Conclusion

Physics of Force-Free Flux Rope

In a force-free flux rope there is no net magnetic stress

$$\mathbf{J} \times \mathbf{B} = 0 \tag{1}$$

We can rewrite (1) as the sum of a magnetic pressure gradient force and the divergence of the magnetic stress tensor

$$-\nabla\left(\frac{B^2}{2\mu_o}\right) + \frac{1}{\mu_o}\nabla \bullet (\mathbf{B}\,\mathbf{B}) = 0 \tag{2}$$

Thus in a rope the outward force of the magnetic pressure is balanced by the inward force of the twisted magnetic field

A force-free rope has no current perpendicular to **B**. Thus we may write

$$\nabla \times \mathbf{B} = \mu_o \mathbf{J} = \alpha \mathbf{B} \tag{3}$$

There is no constraint on the functional form of α. It may vary with distance from the center of the rope. If it is constant, then we take the curl of (3) to obtain

$$\nabla(\nabla \bullet \mathbf{B}) - \nabla^2 \mathbf{B} = \alpha(\nabla \times \mathbf{B}) \tag{4}$$

$$\nabla^2 \mathbf{B} = -\alpha(\nabla \times \mathbf{B})$$

The solutions of this second order equation are the zeroth and first-order Bessel functions [Lundquist, 1950]

$$B_a = B_o\, J_o\,(\alpha r) \tag{5}$$
$$B_p = B_o\, J_1\,(\alpha r)$$
$$B_r = 0$$

If α is constant, the rope is said to be in the Taylor state

In the Taylor state $\frac{J}{B}\left(=\frac{\alpha}{\mu_o}\right)$ is constant

Non-Force Free Cylindrically Symmetric Model

We express the poloidal and axial fields in terms of components

$$B_a = B_1 [\exp \{-(r/\sigma_a a)^m\}]$$

$$B_p = B_o [1 - \exp \{-(r/\sigma_p a)^n\}]$$

that increase and decrease from the center of the rope respectively as exponentials raised to powers n, and m, each with their own independent field strength, exponential scale length and exponent. The two parameters of the Bessel function fit have been replaced by six in the exponential fit.

In practice we have used an "expansion" factor δ to account for asymmetry in the time profile, most probably due to the expansion of the rope as it moves across the observer.

Both models also solve for the orientation of the rope, clock and cone angles, and the impact parameter, the distance of the satellite from the central axis of the rope at closest approach.

Summary and Conclusions

- The Bessel function model of a cylindrical symmetric flux rope is adequate only for force-free ropes in the Taylor state

- The magnetic clouds observed by Pioneer Venus in the 1980's were often not force-free and not in the Taylor state

- Most ropes analyzed had a net outward force if they showed any unbalanced force

- Ropes are not in general cylindrically symmetric. STEREO will help us probe the cross flow extent of such structures and even better understand their dimensions and force balance